

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT3

Branch: B. Tech (All)

Semester : 1

Date : 14/03/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) nth derivative of $y = e^{-x}$ is
 (A) $-e^{-x}$ (B) $(-1)^n e^{-x}$ (C) $(-1)^{n+1} e^{-x}$ (D) none of these
- b) If $y = \sin 2x \cos 2x$ then y_n equal to
 (A) $\frac{1}{2}(4)^n \cos\left(\frac{n\pi}{2} + 4x\right)$ (B) $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 4x\right)$
 (C) $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 2x\right)$ (D) none of these
- c) If $y = \sin^{-1} x$, then x equal to
 (A) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ (B) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$
 (C) $1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \dots$ (D) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$
- d) The series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$ represent expansion of
 (A) $\sin x$ (B) $\cos x$ (C) $\sinh x$ (D) $\cosh x$
- e) $\lim_{x \rightarrow 0} \frac{5^x - 3^x}{x} = \underline{\hspace{2cm}}$
 (A) 2 (B) $\log 2$ (C) $\log 15$ (D) $\log\left(\frac{5}{3}\right)$
- f) $\lim_{x \rightarrow \infty} x^n e^{-ax}$ (n being a positive integer and $a > 0$) = $\underline{\hspace{2cm}}$
 (A) -1 (B) 0 (C) 1 (D) None of these
- g) If $Q = r \cot \theta$, then $\frac{\partial Q}{\partial r}$ is equal to
 (A) $\cot \theta$ (B) $-\cos^2 \theta$ (C) $\cot \theta - r \operatorname{cosec}^2 \theta$ (D) $\frac{1}{2} \cot \theta$



- h) If $u = ax^2 + 2hxy + by^2$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to
 (A) $2u$ (B) u (C) 0 (D) none of these
- i) If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to
 (A) $\frac{\partial f / \partial x}{\partial f / \partial y}$ (B) $\frac{\partial f / \partial y}{\partial f / \partial x}$ (C) $-\frac{\partial f / \partial y}{\partial f / \partial x}$ (D) $-\frac{\partial f / \partial x}{\partial f / \partial y}$
- j) If $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)}$ is equal to
 (A) 1 (B) -1 (C) 0 (D) none of these
- k) If $y = \cos \theta + i \sin \theta$, then the value of $y + \frac{1}{y}$ is
 (A) $2 \cos \theta$ (B) $2 \sin \theta$ (C) $2 \operatorname{cosec} \theta$ (D) $2 \tan \theta$
- l) The number of solutions to the equation $z^2 + \bar{z} = 0$ is
 (A) 1 (B) 2 (C) 3 (D) 4
- m) If A is a non-zero column vector $(n \times 1)$, then the rank of matrix AA^T is
 (A) 0 (B) 1 (C) $n-1$ (D) n
- n) The matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ is given. The eigenvalues of $4A^{-1} + 3A + 2I$ are
 (A) $6, 15$ (B) $9, 12$ (C) $9, 15$ (D) $7, 15$

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) If $y = \frac{x}{x^2 + a^2}$ then find y_n . (5)
- b) Expand $f(x) = \frac{e^x}{e^x + 1}$ in powers of x up to x^3 by Maclaurin's series. (5)
- c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that (4)
- $$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}.$$

Q-3 Attempt all questions (14)

- a) If $y = \sin(m \sin^{-1} x)$ then prove that (5)
- $$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$
- b) Prove that $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$ (5)
- c) Evaluate: $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ (4)

Q-4 Attempt all questions (14)



a) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$ (5)

b) If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$, evaluate $J = \left(\frac{x, y}{u, v} \right)$ and $J' = \left(\frac{u, v}{x, y} \right)$ and hence (5)

verify that $JJ' = 1$.

c) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in powers of $(x-3)$. (4)

Q-5

Attempt all questions

(14)

a) If $u = \sec^{-1} \left(\frac{x^2 + y^2}{x-y} \right)$ then find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. (5)

b) Evaluate: $\lim_{x \rightarrow 0} \frac{a}{x^2} \left[\frac{\sin kx}{\sin lx} - \frac{k}{l} \right]$ (5)

c) Find n^{th} derivative of $\tan^{-1} x$. (4)

Q-6

Attempt all questions

(14)

a) Using the formula $R = \frac{E}{I}$, find the maximum error and percentage of (5)

error in R if $I = 20$ with a possible error of 0.1 and $E = 120$ with a possible error of 0.05 and $R = 6$.

b) Find the continued product of all the values of $\left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^{\frac{3}{4}}$. (5)

c) Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. (4)

Q-7

Attempt all questions

(14)

a) Find the inverse of $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan reduction (5)

method.

b) Find the fourth roots of unity and sketch them on the unit circle. (5)

c) If $\tan(\alpha + i\beta) = x + iy$ then prove that $x^2 + y^2 + 2x \cot 2\alpha = 1$. (4)

Q-8

Attempt all questions

(14)

a) Investigate for what values of λ and μ the equations (5)

$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

b) If $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$ then prove that $\lim_{n \rightarrow \infty} x_1 x_2 x_3 \dots x_n = -1$. (5)

c) Check whether the following set of vectors is linearly dependent or linearly independent: (4)

$(1, 0, 1), (1, 1, 0), (1, -1, 1), (1, 2, -3)$

